

Structural analysis with finite elements

A basic explanation of the finite element method
applied to structural engineering

Paolo Rugarli
Castalia



Published by Thomas Telford Limited, 40 Marsh Wall, London E14 9TP, UK.
www.thomastelford.com

Distributors for Thomas Telford books are
Australia: DA Books and Journals, 648 Whitehorse Road, Mitcham 3132, Victoria

First published 2010

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First Italian edition 'Calcolo strutturale con gli elementi finite' by Paolo Rugarli
ISBN: 88-8184-295-8
EPC Libri Srl, Roma, Italia, 2003

www.icevirtuallibrary.com

A catalogue record for this book is available from the British Library

ISBN: 978-0-7277-4093-9

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Printed and bound in Great Britain by Antony Rowe Ltd, Chippenham

To Roberta, Francesco and Stefano.

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First Italian edition:

Paolo Rugarli
Calcolo strutturale con gli Elementi Finiti
EPC Libri Srl, Rome (Roma), 2003
ISBN 88-8184-295-5

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PREFACE TO THE ENGLISH EDITION

This edition of my work on finite elements is substantially identical to the first Italian edition, but has been improved by some emendations to the text and the correction of a few errors and misprints. I wish to thank, in this regard, my friend and colleague Engineer Giorgio Nieri, who carefully read the text in order to find the errors that had escaped notice in the Italian edition and so allowed them to be corrected here.

The Italian version of the book includes a CD with a limited version (fifty nodes) of the finite element program "SARGON", which I developed over the course of some twenty years of work. Without disparaging the other programs available, I believe that free access to a program which is perfectly functional and can serve as a trial is extremely helpful, both for those who are learning to work with finite elements for the first time and those who already own other programs and wish to try another one. Before undertaking complex models with many degrees of freedom, it is important for beginners to have a clear understanding of how the finite element method works on small models. The English demo version of Sargon is available as shareware at this website: www.steelchecks.com/pro/sr/demo.asp.

I wish to thank the publisher, Thomas Telford, for their willingness to publish this work, and for their openness to new ideas, and in particular I sincerely thank Matthew Lane and Daniel Keirs, who showed an interest in my work from the very beginning.

The present English edition of this volume is the result of the scrupulous translation from the original Italian by Kim Williams, a specialist in the translation of scientific works from Italian to English: the author has found in her a valuable collaborator. To her go my most profound thanks.

Paolo Rugarli
Milan, February 2010

PREFACE TO THE ITALIAN EDITION

This book grew out of the desire to provide an easily understandable guide illustrating some of the principal aspects of the Finite Element Method, in order to facilitate awareness of the considerable power of the tools at the engineer's disposal today. It is designed as an intermediate link between books that provide no theoretical information whatsoever and specialised textbooks: the aim during its composition was to arrive at a clear articulation of the essential core of the problems, without allowing concerns about rigorous formal orthodoxy to obstruct that object. Ironically, in some case a simplification would have required another book (for instance, chapter 3, references to simplified theory, or chapter 11, the in-depth description of basic finite elements), and thus in this context greater formality was required for brevity's sake.

This present volume presumes to be nothing more nor less than an aid. While writing it, I have seen more clearly its limits, and the limits of my effort: I am well aware that works of this kind can easily be accused of oversimplification, approximation, incompleteness and presumption. Be that as it may, it is still necessary and urgent to provide engineers with a text on the finite element method that is less irksome and thorny, and thus of greater advantage to the quality of structural calculations, than is usual.

This book is an effort in that direction.

I wish to thank all of my colleagues who, over the years, through the questions and observations that arose in the course of actual practice, as well as their trust in me, have made it possible for me to examine carefully and better understand many problems which are dealt with here. I also wish to thank Mrs Clea Nardi, who helped with drawing the illustrations. Finally, I wish to thank my school, the Politecnico di Milano.

Milan, July 2003