

Modern structural analysis

Modelling process and guidance

Iain A. MacLeod

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Foreword

This interesting book promotes a new way of looking at structural analysis. It suggests that the ability to work with the model (as distinct from the solution process) is a primary issue which should be formally addressed in practice and in education. The content is focused on modelling issues and I know of no other text which does this so comprehensively.

The early chapters contain much advice necessary to help the reader establish how to formulate a numerical model that might be capable of simulating the performance of the actual structural system under investigation. The later chapters include a good outline of the issues involved in modelling of structures using finite elements. The two case studies given at the end of the book are a good device to put the excellent advice given in the earlier sections into some perspective for the reader.

I found it most useful to have in the same book a reminder of the theoretical basis of the full range of finite element types and a sound method as to how to employ analysis as a reflective tool towards a better understanding of structural behaviour. The rigorous treatment for the process of validation of a model is most enlightening as is that outlined for verification of the results. After all, the iterative process of model validation and output verification are the main activities for gaining a true understanding of structural behaviour.

My own experience working with Buro Happold tells me that robust structural design requires the willingness to develop an understanding of structural behaviour with a questioning mind. In most consulting offices, current practice is to undertake this using finite element models of increasing complexity as understanding of the problem at hand grows. Iain MacLeod describes clearly how to build up this understanding using sensitivity analysis and simplified loadings to test validity against expectations from parallel calculation and modelling experiences. It is argued that risk will be reduced in practice if there is a rigorous analytical process that reflects the realities of current engineering practice in most offices.

Most structures are of a reasonably conventional type and use well tried framing systems. Substantial experience already exists on their likely performance so hand calculations based on structural theory can be done to initiate formulation of the model or to act as a check on the results. However, even advanced classical methods struggle to model the sophistication of load paths in redundant or non-linear structures where individual stiffness, material response and definition of restraint determines structural performance. In this case, I have found that comparison of the output of simplified analytical results with physical models very useful as an addition to classical calculation – as advocated in the second chapter.

The book is thus both a useful reference for the practitioner and a comprehensive learning guide for the student. It builds on the publication by the Institution of Structural Engineers *Guidelines for the Use of Computers for Engineering Calculation* published in 2002. Its carefully constructed content successfully redresses the imbalance in risk between the finite element process based around generally determinate calculation output that has itself been derived from a possibly non-determinate understanding of the actual modelling process. In the Introduction, the author suggests that all structural engineers and all civil engineers who use structural analysis will find the contents of the book to be useful. I think that he is right.

Michael Dickson FStructE
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3 The modelling process

3.1 Overview of the modelling process

3.1.1 General

The process discussed here is basically that advocated in other publications, for example IstructE (2002), MacLeod (1995), NAFEMS (1995, 1999) and ISO 9001 (2000). It tends to be used in a formal way by those who specialise in analysis modelling, and in a tacit way by many practitioners.

In order to reduce risk in analysis modelling a formal modelling process should always be adopted. By *formal* is meant that a written record of the activities of the process should be produced. Reasons for formalising the process include the following:

- it helps to minimise the risks in the use of structural analysis
- it helps to avoid omission of important activities.

Making the process formal provides evidence of the use of good practice should the adequacy of the modelling work be later questioned.

The process described here is for structural analysis contexts but it is directly relevant to any analysis modelling situation (e.g. geotechnical models, hydraulic models, etc.) and can be adapted to other types of model, such as physical models, etc.

3.1.2 Representations of the modelling process

A *determinate* process is one for which there is a unique result. Having decided on a structural analysis model, the solution process provides an unique set of results and hence is determinate. The only part of the modelling process which is determinate is the solution process. A *non-determinate* process does not have a unique solution. All the other activities of the modelling process have non-determinate outcomes and therefore the overall modelling process is non-determinate.

Figure 3.1 and Table 3.1 give different views of the modelling process. Figure 3.1 is a flow diagram of the modelling process: the boxes represent outcomes (no fill for the box) or subprocesses (grey fill for the box). Table 3.1 is another view of the process, one which emphasises the need for acceptance criteria at each stage.

Although these views can be interpreted as implying a linear implementation, the real process is likely to involve much looping back to previous stages – it will not normally be linear. It is not possible to model such non-linearity and therefore Fig. 3.1 and Table 3.1 are not strictly definitions of process but rather are a list (Fig. 3.1) and a matrix (Table 3.1) of activities and outcomes set out in an order in which they normally first occur.

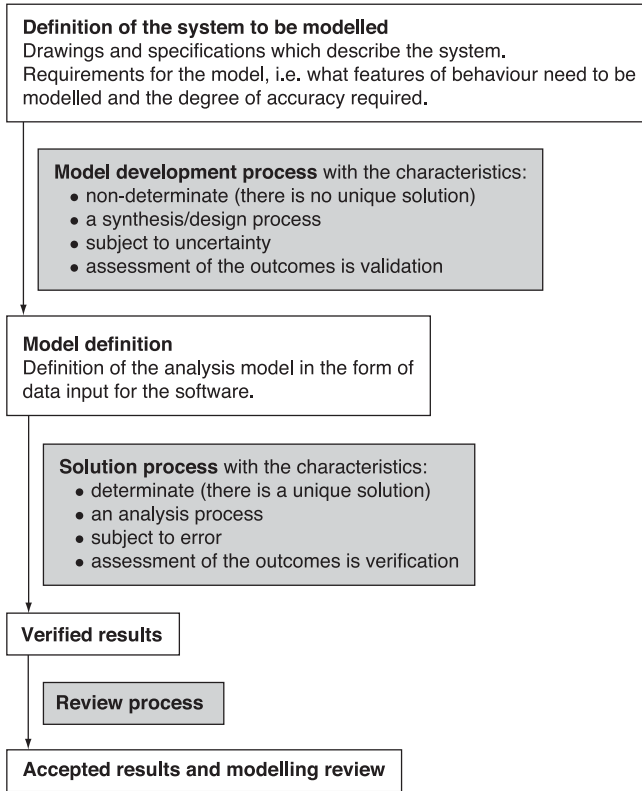


Figure 3.1 The modelling process.

Table 3.1 Modelling process matrix

	A Model development	B Acceptance criteria	C Model assurance
1 Input	Define the system to be modelled		
2 Analysis model	Define the analysis model	Define acceptance criteria	Validate the analysis model
3 Software	Select suitable software	Define acceptance criteria	Software validation and verification
4 Results	Perform calculations to get results	Define acceptance criteria	Results verification
5 Review		Define overall acceptance criteria	Carry our sensitivity analysis Accept or reject the overall solution Produce modelling review document
6 Output	Define the results to be used for design		

Table 3.2 Modelling activities checklist

1	Define the requirements
2	Validate the model
3	Verify the results
4	Review the outcomes

The process activities set out in Fig. 3.1 and Table 3.1 are normally used by those who do structural analysis. What is often not standard is the treatment of some of the activities in a formal way. In particular, the activities listed in Table 3.2 are often not given enough attention or adequately recorded. Attention to these activities can significantly reduce the risk inherent in structural analysis.

3.1.3 Validation and verification

The following definitions are used in this text (IStructE 2002).

- *Validation* is the consideration of whether or not a process is suited to its purpose. The fundamental question in validation is: is the process capable of satisfying the requirements? – or alternatively: is it the right process?
- *Verification* is the consideration of the question: has the process been implemented correctly? – or alternatively: is the process right?

These definitions are in general agreement with those given in ISO 9001 (2000).

3.1.4 Error and uncertainty

In a modelling process, it is necessary to work with the deviations between the benchmark value of a variable and the value that you have. The *benchmark value* is the desired value of the variable. This leads to the following view of the difference between error and uncertainty.

- *Error* is deviation where the benchmark value is ‘exact’ – see Section 2.4.5. It is the result of a determinate process. For example, a set of simultaneous equations normally has a potentially exact solution (although real solutions are always approximations). Similarly, the value of π is potentially exact (although there will always be an error in stating it).
- *Uncertainty* is the situation where there is no unique result against which given values can be compared. The outcomes from a non-determinate process are subject to uncertainty, as are the values of material constants. For example, there is no unique value for the value of Young’s modulus of concrete (Section 7.2.4); the value depends on how it is measured, and even if the same method is used each time there will be differences in the results for every measurement.

In verification, error tends to be the main consideration, and in validation, uncertainty tends to dominate. Appreciation of the difference between error and uncertainty is important because the tolerance in acceptability is likely to be much greater for uncertainty than for error, as shown in the following examples.

- In defining stiffness for a soil, a deviation (uncertainty) of 10% could be satisfactory.
- In the solution of the system equations in a finite element model, an error check for equilibrium or symmetry should compare up to the last significant figures in the output value. Normal double precision arithmetic for finite element solutions gives 13 significant figures, so the sought accuracy is of the order of 10^{-12} – see example in Section 12.1.6.

3.2 Defining the system to be modelled

The definition of the system to be modelled is sometimes called the *engineering model* (IStructE 2002). Items to be considered include the following:

- *Portrayal of the engineering system to be modelled* – this would be mainly in the form of drawings, sketches and specifications.
- *Requirements of the model* – it is essential to define the outcomes that are required from the modelling activity. Typical objectives of modelling are to predict:
 - stresses or stress resultants
 - failure conditions
 - short-term deformations
 - long-term deformations
 - instability conditions
 - dynamic characteristics.

One of the requirements should be a statement of the desired accuracy of the results. This will depend on the context and, especially, on the degree of risk involved, both with respect to the consequences of failure and to the degree of innovation involved.

3.3 The model development process

3.3.1 Conceptual and computational models

The analysis model is the mathematical representation of the system. It has two components (IStructE 2002).

- The *conceptual model* is defined in terms of material behaviour, loading, boundary conditions, etc. For example, in the analysis of a floor slab the conceptual model could involve linear elastic material behaviour, thin plate bending theory and point supports.
- The *computational model* incorporates the means of achieving a solution. In the case of the floor slab model, the computational model could be based on a specific plate bending finite element mesh (Section 6.3.4) or a grillage model (Section 6.3.6). In some cases the boundary conditions may be part of the computational model; for example, an elastic half-space conceptual model can be reduced to a finite size in the computational model by imposing boundary conditions – see Fig. 8.9. In some situations, for example for elastic frame analysis, computational modelling issues may seldom need to be addressed.

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